PolicePrep Comprehensive Guide to Canadian Police Officer Exams

Teaching Material
Math

Addition

$$\begin{array}{r} 7 \\ +\quad 5 \\ \hline 12 \end{array} \qquad\qquad 7 + 5 = 12$$

The above two equations have the same value and are very straightforward. It is important to know that the order of numbers does not make a difference in addition (or multiplication). For example:

$$\begin{array}{r} 6 \\ +\ 3 \\ \hline 9 \end{array} \qquad \text{same} \qquad \begin{array}{r} 3 \\ +\ 6 \\ \hline 9 \end{array}$$

$$243 + 716 = 959$$

same

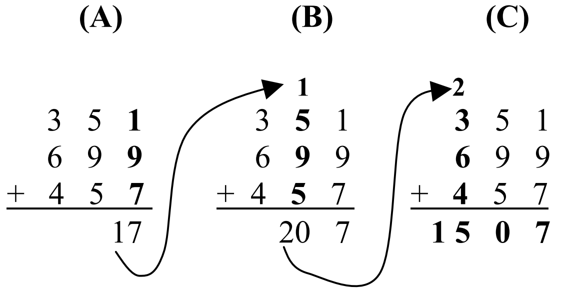
$$716 + 243 = 959$$

Some complications arise when larger numbers are used and you need to carry numbers.

Note: When you see a math problem laid out horizontally, as in the box immediately above, rearrange the numbers so that they are vertical (on top of each other) to make the addition easier to do.

Example:

$$\begin{array}{r} 3\ 5\ 1 \\ 6\ 9\ 9 \\ +\,4\ 5\ 7 \\ \hline \end{array}$$



(A)
Start by adding up the numbers in the right most column. The result is 17. The seven remains but the one is carried over to be added to the next column of numbers.

(B)
The same rules apply to the sum 20 in the second column. The 0 remains in the second row, while the 2 is carried over to the column to the left to be added.

(C)
The final column is then added and the answer is recorded.

Subtraction

$$\begin{array}{r} 8 \\ - \quad 3 \\ \hline 5 \end{array} \qquad\qquad 8 - 3 = 5$$

The above two equations have the same value and are very straightforward. It is important to know that the order of numbers is significant in subtraction (and division). Different ordering will result in different answers. For example:

$$\begin{array}{r} 18 \\ - \quad 3 \\ \hline 15 \end{array} \qquad \text{different} \qquad \begin{array}{r} 3 \\ -\ 18 \\ \hline -\ 15 \end{array}$$

712 - 245 = 467
different
245 - 712 = - 467

Some complications arise when larger numbers are used and you need to carry numbers.

Example:

$$\begin{array}{r} 7\,4\,3 \\ -\ 5\,8\,9 \\ \hline \end{array} \qquad\qquad \overset{\textbf{(A)}}{\begin{array}{rccc} & & 3 & \\ & 7 & \cancel{\mathbf{4}} & {}_{1}\mathbf{3} \\ - & 5 & 8 & \mathbf{9} \\ \hline & & & 4 \end{array}} \qquad\qquad \overset{\textbf{(B)}}{\begin{array}{rccc} & 6 & 13 & \\ & \cancel{7} & \cancel{\mathbf{4}} & 3 \\ - & 5 & \mathbf{8} & 9 \\ \hline & 1 & 5 & 4 \end{array}}$$

(A)
The first task is to subtract the right most column. Because 9 is larger than 3, a unit has to be borrowed from the column to the left. The 4 in the middle column is reduced to 3, and the one is added to the right column, making the first row 13 - 9 = 4.

(B)
The second task is to subtract the second column. The same process is repeated. Borrow a 1 from the left column to allow the subtraction. The top number in the left column becomes 6, while the top number in the centre column becomes 13. 13 - 8 = 5. The left column would then be subtracted. 6 - 5 = 1.

Note: If subtracting more than 2 numbers, you cannot stack the numbers as you would in addition. Instead, work from the first subtraction to the last, two numbers at a time.

Multiplication

$$\begin{array}{r} 8 \\ \underline{\text{x} \quad\; 6} \\ 48 \end{array} \qquad\qquad 8 \text{ x } 6 = 48$$

The above two equations have the same value and are very straightforward. It is important to know that the order of numbers makes no difference in multiplication (or addition). For example:

$$\begin{array}{r} 7 \\ \underline{\text{x} \quad\; 8} \\ 56 \end{array} \qquad\qquad \text{same} \qquad\qquad \begin{array}{r} 8 \\ \underline{\text{x} \quad\; 7} \\ 56 \end{array}$$

245 x 233 = 57,085
same
233 x 245 = 57,085

Multiplication, simply put, is adding groups of numbers. For instance, in the above example, the number 8 is being added six times.

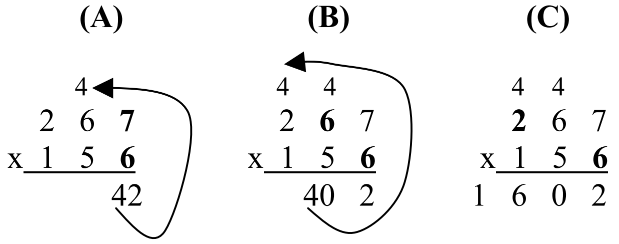
8 x 6 = 48 8 + 8 + 8 + 8 + 8 + 8 = 48	7 x 7 = 49 7 + 7 + 7 + 7 + 7 + 7 + 7 = 49
9 x 5 = 45 9 + 9 + 9 + 9 + 9 = 45	6 x 3 = 18 6 + 6 + 6 = 18

It will be difficult to pass an exam if you have to calculate all simple multiplication in this manner. You should memorize the basic multiplication tables for 1 through 12. Review the multiplication table in this book.

Some complications arise when larger numbers are used and you need to carry numbers.

Example:

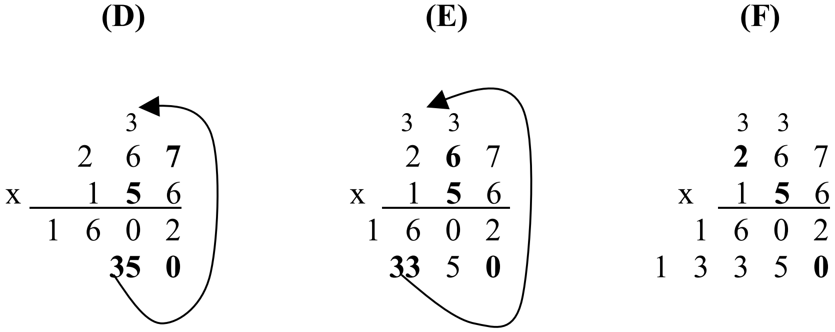
$$\begin{array}{r} 267 \\ \underline{\times\,156} \end{array}$$



(A)
Begin by multiplying out the right row. The 2 is recorded in the right column and the 4 is transferred to the middle column and recorded as above.

(B)
The second step is to multiply the 6 in the middle column. 6 x 6 = 36. The 4 that was carried over from step A has to be added to the 36. The result is 40 and the 0 is recorded in the middle column. The four is then carried forward to left column as in step A.

(C)
The 6 then has to be multiplied to the left digit on the top number. 6 x 2 = 12. The four that was carried over from step B is added to the 12. The result is 16 and recorded as shown.



(D-F)
The next steps are to multiply the second digit in the bottom row (the 5) to each of the top digits. The 5 is multiplied to the 7, the 6 and the 2. The process is the same as steps A - C. If the number is 10 or larger the number is carried over, as above, and added to the next multiplication.

It is important to remember that the next multiplication set has to be recorded on the line below and lined up starting in the next column. Place a zero in the right column to ensure the digits line up properly

(G)

$$\begin{array}{rrrrrr} & & & 2 & 6 & \mathbf{7} \\ \text{x} & & & \underline{\mathbf{1}} & \underline{5} & \underline{6} \\ & & 1 & 6 & 0 & 2 \\ & 1 & 3 & 3 & 5 & \mathbf{0} \\ & & & \mathbf{7} & \mathbf{0} & \mathbf{0} \end{array}$$

(H)

$$\begin{array}{rrrrrr} & & & 2 & \mathbf{6} & 7 \\ \text{x} & & & \underline{\mathbf{1}} & \underline{5} & \underline{6} \\ & & 1 & 6 & 0 & 2 \\ & 1 & 3 & 3 & 5 & \mathbf{0} \\ & & \mathbf{6} & 7 & \mathbf{0} & \mathbf{0} \end{array}$$

(I)

$$\begin{array}{rrrrrr} & & & \mathbf{2} & 6 & 7 \\ \text{x} & & & \underline{\mathbf{1}} & \underline{5} & \underline{6} \\ & & 1 & 6 & 0 & 2 \\ & 1 & 3 & 3 & 5 & \mathbf{0} \\ & \mathbf{2} & 6 & 7 & \mathbf{0} & \mathbf{0} \end{array}$$

The next steps are to multiply the left digit in the bottom number by each of the digits in the top number. The same process is used as outlined above if numbers have to be carried over.

Lining up of the digits is also necessary at this stage. Because you are multiplying from the hundreds column (the left most) you begin recording the answer in the hundreds column. Follow the same procedure as outlined above. Fill in the first two columns with zeros.

$$\begin{array}{rrrrrr} & & & 2 & 6 & 7 \\ \text{x} & & & \underline{1} & \underline{5} & \underline{6} \\ & & \mathbf{1} & \mathbf{6} & \mathbf{0} & \mathbf{2} \\ & \mathbf{1} & \mathbf{3} & \mathbf{3} & \mathbf{5} & \mathbf{0} \\ + & \underline{\mathbf{2}} & \underline{\mathbf{6}} & \underline{\mathbf{7}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} \\ & \mathbf{4} & \mathbf{1} & \mathbf{6} & \mathbf{5} & \mathbf{2} \end{array}$$

The final step is to add up the three numbers that were multiplied out. Treat the addition of these three numbers exactly as you would a regular addition problem. If you failed to line the numbers up properly, you will wind up with an incorrect answer. 41,652 is the final answer.

Note: Because complex multiplication questions (like the one above) involve addition, make sure you have a firm grasp of the addition section before trying to tackle multiplication.

Things to Watch For

Watch out for a multiplication question where the first digit in the bottom number is a zero, or where there are zeros in the equation. You still have to properly line up the digits. Note the highlighted zeros.

$$\begin{array}{ccccc} & & 3 & 4 & 5 \\ \times & & & 5 & 0 \\ \hline 1 & 7 & 2 & 5 & \mathbf{0} \end{array}$$

Remember that zero multiplied by any other number is zero. In this situation you begin multiplying with the 10's column (the 5). Because you are multiplying from the 10's column, you begin recording your answer there. Place a zero in the first column.

$$\begin{array}{cccc} & & \mathbf{3} & \\ & 6 & \mathbf{0} & 9 \\ \times & & & \mathbf{4} \\ \hline 2 & 4 & \mathbf{3} & 6 \end{array}$$

When the four is multiplied to the 0, the result is 0. The number, which is carried over from multiplying 9 x 4 has to be added to 0, which results in the highlighted answer - 3.

$$\begin{array}{ccccccc} & & & & 4 & 5 & 2 \\ & & \times & & 3 & 0 & 9 \\ \hline & & & 4 & 0 & 6 & 8 \\ + & 1 & 3 & 5 & 6 & \mathbf{0} & \mathbf{0} \\ \hline & 1 & 3 & 9 & 6 & 6 & 8 \end{array}$$

In this situation there is no need to multiply the bottom ten's digit out, as the result will equal 0. You must, however, properly line up the numbers. Because the 3 is in the hundred's column, you must begin recording your answer in the hundred's column. That is why there are two highlighted zeros.

Multiplication Tables

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Use of the Table

To use this table, take a number along the top axis and multiply it by a number along the side axis. Where they intersect is the answer to the equation. An example of this is 7 x 3. If you find 7 on the side axis and follow the row until you reach the 3 column on the top axis, you will find the answer – 21.

Look for simple patterns to assist your memorization efforts. For example:

Whenever 10 is multiplied to another number, just add a zero.

10 x 3 = 30   10 x 7 = 70
10 x 10 = 100   10 x 12 = 120

Whenever 11 is multiplied by a number less than 9, just double the digit 11 is multiplied by.

11 x 3 = 33   11 x 5 = 55
11 x 7 = 77   11 x 9 = 99

One multiplied by any other number is always equal to that number.

1 x 1 = 1   1 x 4 = 4
1 x 8 = 8   1 x 12 = 12

Zero multiplied to any number is always zero.

0 x 10 = 0   0 x 3 = 0

Nine multiplied by any number less than 11 adds up to 9.

9 x 3 = 27   (2 + 7 = 9)
9 x 9 = 81   (8 + 1 = 9)

Division

$6 / 3 = 2$ $\qquad$ $6 \div 3 = 2$

$\frac{6}{3} = 2$ $\qquad$ $\begin{array}{r} 2 \\ 3\overline{)6} \end{array}$

The above equations have the same values and are very straightforward. It is important to know that the order of the numbers is significant in division (and subtraction). Different ordering of numbers will result in different answers. For example:

$10 / 5 = 2$ different $5 / 10 = 0.5$

$15 \div 5 = 3$ different $5 \div 15 = 0.33$

$\frac{100}{10} = 10$ different $\frac{10}{100} = 0.1$

$\begin{array}{r} 5 \\ 10\overline{)50} \end{array}$ different $\begin{array}{r} 0.2 \\ 50\overline{)10} \end{array}$

Simply put, division determines how many times a number will fit into another. Picture an auditorium with 100 chairs available. Several schools want to send 20 students to see a play in the auditorium. Now you need to determine how many schools can attend the play. This will require division.

By dividing 100 by 20 ($100 \div 20$) you come up with the number 5. Five schools can send 20 students to attend the play.

Long Division

$6 / 3 = ?$

$\boxed{\begin{array}{r} ? \\ 3\overline{)6} \end{array}}$

$6 \div 3 = ?$

$\frac{6}{3} = ?$

When performing long division, it is important to organize the information as is seen in the centre square. You have to understand how the different formats for division are transferred into the format seen above.

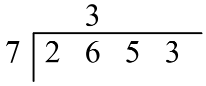
Example

2653 ÷ 7 = ?

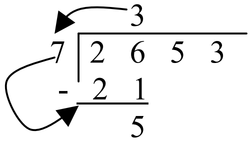
$$7\overline{)\;2\quad 6\quad 5\quad 3}$$

In order to answer a division question on paper, you must place the equation in the proper format. After this is accomplished you can begin to solve the problem.

(A)



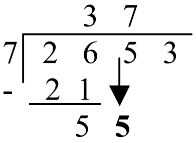
(B)



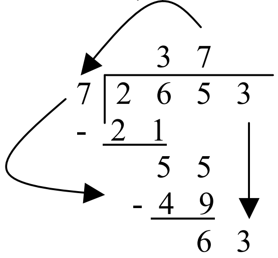
(A)
The first step is to focus on the highlighted area of the number under the bracket. You have to work with a number that is larger than the dividing number (7). Because 2 is smaller than 7, you have to work with 26. Ask yourself how many times you can multiply 7 without going over 26. If you count by 7's (7, 14, 21, 28) you'll realize that 3 is the most times that 7 will fit into 26.

(B)
With the information you have in section A, you now have to perform a simple multiplication. Take the top number (3) and multiply it by the dividing number (7). The answer is placed below 26 and then subtracted from the digits you were working with. (26 - 21 = 5) Make sure you keep the numbers in the proper columns. (If, after subtracting, the answer is greater than the dividing number, you need to start again using a larger top number.)

(C)



(D)



(C)
After subtraction, bring down the next digit to sit beside the solution. This becomes your new number to work with (55). Then repeat step A using this number. Determine how many times you can multiply 7 without exceeding 55. Place this digit above the next digit in the question on top of the bracket.

(D)
Next repeat step B. Multiply out the 7's and record your answer below the 55. Subtracting the numbers results in 6. Continue to work the same pattern, and bring down the next digit in the question to determine a new number to work with.

(E)

$$
\begin{array}{rl}
 & 379 \\
7 & \overline{)2653} \\
- & \underline{21} \\
 & 55 \\
- & \underline{49} \\
 & 63 \\
- & \underline{63} \\
 & 0
\end{array}
$$

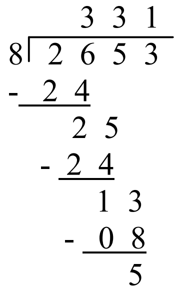
(E)
The final steps in the process are to repeat the process. Determine how many times you can multiply 7 without going over 63. You can do this 9 times. When you multiply it out and subtract the result is 0. The answer to the question is shown above.

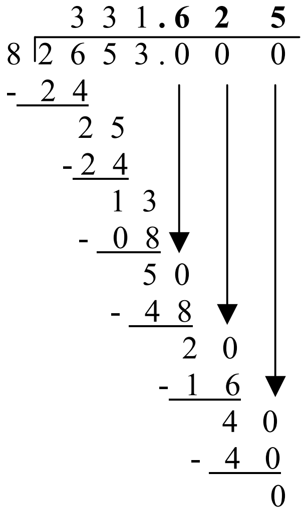
$$2653 \div 7 = 379$$

Decimals

There are times when you are dividing a number and, after the final subtraction, there is a value left over. This is a remainder. When this happens, you can choose whether or not to continue calculating the number. If you continue, 1 or more decimal points will be introduced.

Example:



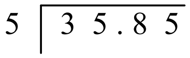


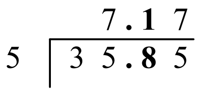
You must follow the same procedure with decimal places as you would with regular long division. Ensure that the digits are properly lined up, and continue adding 0's after the decimal places in the equation.

Decimals and Whole Numbers

You may be required to solve division problems with decimals already in place. Below are two examples of decimals occurring in division questions.

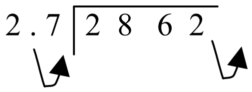
Example 1

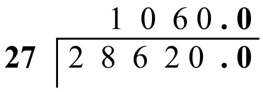




To answer the question correctly, you have to place the decimal point in the answer directly above the decimal point in the question.

Example 2





When a decimal point is found in the denominator (the number of parts into which the whole is divided – bottom number of a fraction), then you must eliminate it before answering the question. This is achieved by shifting the decimal point however many spaces to the right it takes to create a whole number, in this example one space. This has to be matched by shifting the decimal place in the numerator (the number to be divided – top number of a fraction) by one space as well. If the numerator is a whole number, shift the decimal point right by adding a zero, as in the example above.

Example 3

When a decimal point is found in both the numerator and the denominator you must combine both steps. First, you must eliminate the decimal place in the denominator, as in example 2. Then you have to ensure that the new decimal place lines up, as in example 1.

Hints

Long division becomes more complicated with higher numbers, especially higher denominators.

$$\begin{array}{r} \mathbf{0}\,\mathbf{0}\,4\,5 \\ 67\overline{)\,3\,0\,1\,5} \\ \underline{-\,2\,6\,8} \\ 3\,3\,5 \\ \underline{-\,3\,3\,5} \\ 0 \end{array}$$

Using 0's to Line up Numbers

67 will not fit into 3, or 30. You will therefore have to work with 301. By placing 0's above the 3 and the 0, (highlighted), you will not make any errors with improperly aligned numbers.

Rounding Up

Determining how many times 67 will fit into 301 can be a difficult task. It may help to round 67 up to 70. By counting 70 four times, you will reach 280. Five times equals 350, which exceeds 301. Four is the best guess, and by multiplying it out, using 67 you are proven correct.

Disregarding Decimals

The majority of the answers on a test will not require decimals. If your calculation of an equation gives you an answer with decimals, but none of the optional answers have decimals, stop calculating. Make a selection from the available options, or consider that you made a mistake. Quickly check your work, but don't spend too much time on one question that's causing you problems. Move onto the next question.

Zeros and Ones

Any time zero is divided by any other number the answer is 0.

0 / 3 = 0

0 ÷ 25 = 0

$\frac{0}{99} = 0$

$\begin{array}{r} 0 \\ 0\overline{\smash{)}\,99} \end{array}$

It is impossible for a number to be divided by 0. It is indefinable.

9 / 0 = undefined

77 ÷ 0 = undefined

$\frac{66}{0}$ = undefined

Any number divided by 1 is equal to itself.

3 / 1 = 3

55 ÷ 1 = 55

$\frac{1{,}297}{1} = 1{,}297$

$\begin{array}{r} 38 \\ 1\overline{\smash{)}\,38} \end{array}$

Place Value

It is important to maintain proper place value of digits when performing mathematical calculations. You must be able to convert written numbers into digits. For example:

Two million, forty thousand and two	2,040,002
One and a half million	1,500,000
Ten thousand and ten	10,010

You can practice place value questions by answering questions such as the ones below:

a) Write a number that is 100 more than 4, 904.
b) Write a number that is 1000 less than 478, 243.
c) What number is one more than 9,999?
d) What is the value of 5 in the number 241, 598?
e) What figure is in the ten thousands place in 4,365,243?
f) What number is 30,000 less than 423,599?

The answers are listed below.

Place value is important when lining up numbers for addition and subtraction questions. For example:

$$15 + 1043 + 603 + 20,602 = \qquad \begin{array}{r} 20,602 \\ 1,043 \\ 603 \\ \underline{15} \\ 22,263 \end{array}$$

$$13.09 + 0.4 + 206 + 0.002 = \qquad \begin{array}{r} 206.000 \\ 13.090 \\ 0.400 \\ \underline{0.002} \\ 219.492 \end{array}$$

One of the most common errors is failing to place digits correctly under one another, which often occurs when trying to calculate these problems in your head.

Answers to practice questions.

a) 5,004
b) 477,243

c) 10,000
d) 500

e) 6
f) 393,599

Make sure you are comfortable with the proper names for the location of digits in a number.

1, 234, 567.890

1 = millions column
2 = hundred thousands column
3 = ten thousands column
4 = thousands column
5 = hundreds column
6 = tens column
7 = ones column
8 = tenths column
9 = hundredths column
0 = thousandths column

Order of Operations

The following rules have to be obeyed while working with mathematical equations. There is an order to how numbers are manipulated and worked on.

B E D M A S

You should memorize this acronym, as it tells you how to proceed with an equation.

1) **B** – Brackets

   You must perform all mathematical calculations that occur within brackets before any other calculation in the equation.

2) **E** – Exponents

   After calculations within brackets are handled, you have to perform any calculations with exponents next.

3) **D / M** – Division and Multiplication

   Division and multiplication components are next. These are handled in the order they appear reading from left to right.

4) **A / S** – Addition and Subtraction

   The final calculations are individual addition and subtraction questions, which are performed in the order they appear reading from left to right.

The best way to understand this process is to work through several problems.

Example 1:		
6 + 5 x 3 – 7	Step 1: Multiplication	5 x 3 = 15
6 + 15 –7	Step 2: Addition	6 + 15 = 21
21 – 7	Step 3: Subtraction	21-7 = 14
Example 2:		
14 – 7 + 18 ÷3	Step 1: Division	18 ÷ 3 = 6
14 –7 + 6	Step 2: Subtraction	14 – 7 = 7
7 + 6	Step 3: Addition	7 + 6 = 13

Example 3:

7 + (15 – 6 x 2)	Step 1: Brackets	6 x 2 = 12
7 + (15 – 12)	Remember to follow the order of operation within the brackets. (Multiply before subtracting.)	15 – 12 = 3
7 + 3	Step 2: Addition	7 + 3 = 10

Example 4:

$2 (2 + 5)^2$	Step 1: Brackets	2 + 5 = 7
$2 (7)^2$	Step 2: Exponents	$7^2 = 7 \times 7 = 49$
2 (49)	Step 3: Multiplication	2 x 49 = 98

Remember that two numbers separated only by brackets are multiplied together (a bracket = x.)   2 (6) = 6 x 2

Practice Questions

Try these practice questions to see if you are comfortable with mathematical order of operation. The final answers are listed below.

a) 7 – 4 + 6 x 8 ÷2

b) 14 + 8 (6 – 3)

c) $30 – 3(5 – 2)^2$

d) (5 – 1) (4 + 7)

e) $75 – (6 \div (2+1))^2$

f) $10^2 - 10 + 3^2$

g) (10 + 3) x 2 + 6(5-2)

h) $17 + 6^2 (18 \div 9)$

i) 4 (5+2-3+6)

j) 10 (6 + (15 – (10-5)))

Answers

a) 27

b) 38

c) 3

d) 44

e) 71

f) 99

g) 44

h) 89

i) 40

j) 160

Grouping Like Terms

You will come across mathematical problems where you have to group like terms together. Examples of this are very common with money. Whenever you are adding sums of money, there is no need to continually restate the same denominations. Below is an example of an equation adding up a suspect's money:

Denomination	**# of Bills**
$50	4
$20	3
$10	4

One means of calculating the total value of money seized is to individually add up all of the bills.

50 + 50 + 50 + 50 + 20 + 20 + 20 + 10 + 10 + 10 + 10

However, there is an easier and more orderly way of writing and working with this equation. Here is the statement rewritten separating the like terms.

(50 + 50 + 50 + 50) + (20 + 20 + 20) + (10 + 10 + 10 + 10)

Instead of adding all of the $50 bills together you can count the number of 50's and multiply that number by the value.

50 + 50 + 50 + 50 = 4 x 50 or 4 (50)
20 + 20 + 20 = 3 x 20 or 3 (20)
10 + 10 + 10 + 10 = 4 x 10 or 4 (10)

The statement can then be written more clearly as: 4(50) + 3 (20) + 4 (10)

Remember that it doesn't matter what order the terms are in, so long as they remain together. The above equation could be restated any of the following ways:

3(20) + 4(50) + 4(10)   20(3) + 50(4) + 10(4)

20(3) + 10(4) + 50(4)   4(10) + 3(20) + 4(50)

Like terms can occur in any addition question. It doesn't have to be a monetary question. Any time you see two or more of the same number in an addition problem, they can be combined.

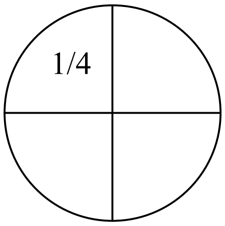
5 + 6 + 3 + 5 + 2 + 6 + 5 = 3(5) + 2(6) + 3 + 2

75 + 63 + 75 + 63 + 75 = 3(75) + 2(63)

5 + 5 + 5 + 5 + 5 + 4 = 5(5) + 4

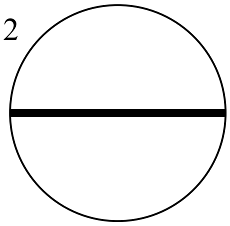
Fractions

A fraction is simply a part of a whole thing. The example below is of a circle divided into four pieces. Each segment represents ¼ of the circle.

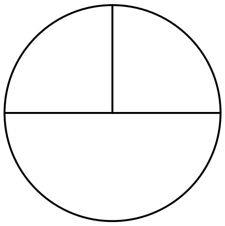


In each of the circles below, the same area is represented, but the area is divided into different numbers of equal parts.

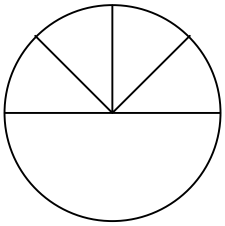
1 / 2



2 / 4

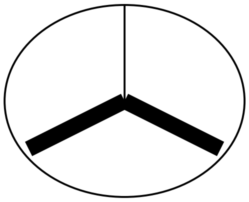


4 / 8

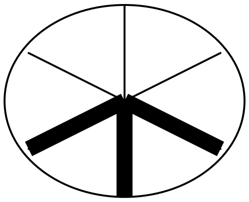


This diagram demonstrates that the fractions 1/2, 2/4 and 4/8 represent the same quantity.

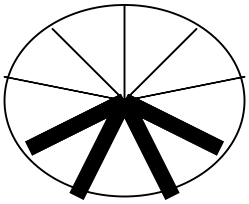
1 / 3



2 / 6



3 / 9



The fractions 1/3, 2/6 and 3/9 are equivalent. You can determine fractions of equivalent value by multiplying both the numerator and the denominator of the fraction by the same number.

$$\frac{1 \text{ x } 7}{3 \text{ x } 7} = \frac{7}{21} \qquad \text{thus} \qquad \frac{7}{21} = \frac{1}{3}$$

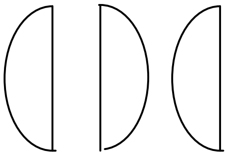
A similar rule holds when dividing the numerators and denominators of fractions. This is necessary to reduce fractions to their lowest form.

$$\frac{5 \text{ divided by } 5}{15 \text{ divided by } 5} = \frac{1}{3}$$

Improper Fractions

When a fraction has a larger numerator than denominator then the fraction is larger than one. The diagram below illustrates an example of improper fractions.

3 / 2 = 1 1/2



Adding and Subtracting Fractions

Whenever you are adding or subtracting fractions, you have to ensure that the denominators of the fractions are the same. For example:

$$\frac{1}{2} + \frac{6}{8} \text{ does not equal } \frac{7}{10}$$

By multiplying both the denominator and the numerator of 1/2 by 4, you will be able to add the fractions together. 1 / 2 becomes 4 / 8.

$$\frac{4}{8} + \frac{6}{8} = \frac{10}{8} = \frac{5}{4}$$

When you are adding and subtracting fractions, you also maintain the same denominator, and add or subtract the numerator.

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{3}{18} + \frac{12}{18} = \frac{15}{18} = \frac{5}{6}$$

$$\frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}$$

$$\frac{7}{8} + \frac{5}{8} = \frac{12}{8} = 1\,\frac{1}{2}$$

Multiplying Fractions

When multiplying fractions, there is no need to find a common denominator. Simply multiply the two top numbers and then multiply the two bottom numbers. Multiplying two fractions together (other than improper) will result in a fraction that is smaller than the original numbers.

$$\frac{4}{5} \times \frac{3}{4} = \frac{12}{20} = \frac{3}{5}$$

$$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$$

$$\frac{3}{4} \times \frac{7}{18} = \frac{21}{72} = \frac{7}{24}$$

$$\frac{3}{2} \times \frac{4}{5} = \frac{12}{10} = 1\,\frac{1}{5}$$

Dividing Fractions

Division with fractions is very similar to multiplying with fractions.

12 divided by 12 = 1	12 goes into 12 once
12 divided by 6 = 2	6 goes into 12 twice
12 divided by 4 = 3	4 goes into 12 three times
12 divided by 3 = 4	3 goes into 12 four times
12 divided by 2 = 6	2 goes into 12 six times
12 divided by 1 = 12	1 goes into 12 twelve times
12 divided by 1/2 = 24	1/2 goes into 12 twenty four times

This is logical when you think about the statement on the right. Whenever you are dividing by a fraction you have to multiply one fraction by the reciprocal of the other. That is, when you divide one fraction by another, you have to multiply one fraction by the inverse of the other. For example:

$$\frac{1}{2} \div \frac{6}{7} = \frac{1}{2} \text{ x } \frac{7}{6} = \frac{7}{12}$$

$$\frac{3}{4} \div \frac{4}{5} = \frac{3}{4} \text{ x } \frac{5}{4} = \frac{15}{16}$$

$$1\frac{3}{4} \div \frac{4}{5} = \frac{7}{4} \text{ x } \frac{5}{4} = \frac{35}{16} = 2\frac{3}{16}$$

Whenever dividing mixed fractions (1 1/2, 2 3/4 etc) you must use improper fractions (3/2, 11/4 etc).

Percentages

It is important to have a solid background in decimals and fractions before you try to handle percentage questions. Percentages are simply fractions. Per means "out of" and cent means "a hundred". Percentages are fractions with 100 as a denominator. They are often noted with this sign: %.

10 % means 10 out of 100 or $\frac{10}{100}$

13 % means 13 out of 100 or $\frac{13}{100}$

100 % means 100 out of 100 or $\frac{100}{100}$

100% means everything. 100% of your salary is your whole salary. You simply follow the same rules of conversion from fractions to decimals for calculating percentages. Simply move the decimal points two places to the left to convert percents to decimals. This is essentially dividing the percentage by 100.

Example: 7 5 % = 0 . 7 5
↶↶
8% = 0 . 0 8
↶↶
5 3 . 5 % = 0 . 5 3 5
↶↶
2 0 8 % = 2 . 0 8
↶↶

Any percent larger than 100% indicates more than the whole. For example:

A man's stock portfolio is worth 125% of what it was a year ago. This means that the stocks are now worth 25% more. If his stocks were worth $500 last year, they would be worth:

$500 x 125% =

$$\begin{array}{r} 500 \\ \underline{\times\ 1.25} \\ \$\ 625 \end{array}$$

Percentages with Fractions

Some questions you encounter may incorporate percentages and fractions. Examples include 2 1/2 % or 33 1/3 %. In order to deal with these problems, you must first convert the percentages to improper fractions.

2 1/2 = 5 / 2   33 1/3 = 100 / 3

After this step you simply carry out the division question.

$$\begin{array}{r} 2.5 \\ 2\overline{)5.0} \\ \underline{4} \\ 10 \\ \underline{10} \\ 0 \end{array} \qquad \begin{array}{r} 33.33 \\ 3\overline{)100.00} \end{array}$$

Once you have the decimal equivalent of the percentage, you then follow the same rules that apply to a regular percentage. Divide the number by 100 or, more simply, move the decimal to the left twice. Thus:

2 1/2% = 0.025   33 1/3% = 0.3333

Percentages You Should Memorize

25% = 1 / 4 = 0.25

50% = 1 / 2 = 0.5

75% = 3 / 4 = 0.75

100% = 4 / 4 = 1.00

33 1/3 % = 1 / 3 = 0.333

66 2/3 % = 2 / 3 = 0.666

10% = 1 / 10 = 0.1

20% = 1 / 5 = 0.2

40% = 2 / 5 = 0.4

60% = 3 / 5 = 0.6

80% = 4 / 5 = 0.8

Decimal / Fraction Conversion Instruction

Fraction to Decimal

There are many situations where you will have to convert fractions to decimals. Decimals are often easier to work with. Changing fractions to decimals is simply a division problem. All you have to do is take the numerator and divide it by the denominator.

Examples:

$$\begin{array}{rl} & \;0.5 \\ 1/2 = 2 & \overline{)\,1.0} \\ - & \underline{\;1.0} \\ & \quad 0 \end{array}$$

$$\begin{array}{rl} & \;0.8 \\ 4/5 = 5 & \overline{)\,4.0} \end{array}$$

$$\begin{array}{rl} & \;0.333 \\ 1/3 = 3 & \overline{)\,1.000} \\ - & \underline{0.9} \\ & 0.10 \\ & \;\;-\underline{09} \\ & \quad\; 010 \\ & \quad\;-\underline{09} \\ & \qquad\;\; 1 \end{array}$$

Mixed Fractions

Mixed fractions have to first be converted to improper fractions before they can be converted to decimals. Multiplying the whole number by the denominator and adding the numerator will achieve this. As soon as the improper fraction is found, you calculate the decimal in the same way as above.

Example 1

$$3\frac{1}{2} = \frac{7}{2} \qquad \begin{array}{rl} & 3.5 \\ 2 & \overline{)\,7.0} \end{array}$$

Multiply 3 by 2, and then add 1. This is the new numerator, and the denominator remains the same.

Example 2

$$2\frac{5}{6} = \frac{17}{6} \qquad \begin{array}{rl} & 2.833 \\ 6 & \overline{)\,17.000} \end{array}$$

Decimal to Fraction

When converting decimals to fractions, place value is extremely important. The first decimal point to the right of the decimal point is the tenths, followed by the hundredths, thousandths, etc. All you have to do is properly line up the place value with the proper denominator.

0.1 is a way of writing $\frac{1}{10}$

0.01 is a way of writing $\frac{1}{100}$

and

0.6 is a way of writing $\frac{6}{10}$

0.78 is a way of writing $\frac{78}{100}$

There is one zero in the denominator for every place to the right of the period in the original decimal.

Exponents

Exponents indicate how many times a number should be multiplied by itself. If a number is raised to the power of 2, the number should be multiplied by itself twice. If the number is raised to the power of 6, the number should be multiplied by itself 6 times.

$2^2 = 2 \times 2 = 4$

$2^3 = 2 \times 2 \times 2 = 8$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

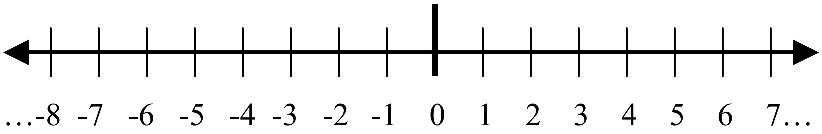
$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$7^2 = 7 \times 7 = 49$

$5^4 = 5 \times 5 \times 5 \times 5 = 625$

Positive and Negative Integers

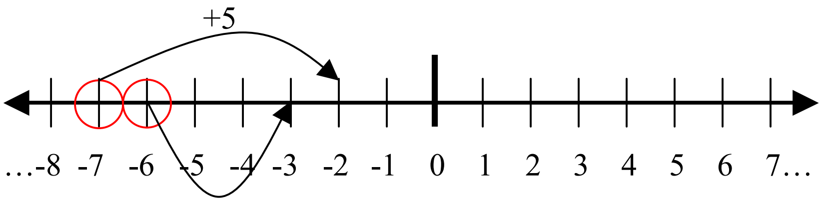
You must have an understanding of positive and negative integers and how they react when they are added, subtracted, multiplied and divided by each other. Look at the number line below. Positive integers exist to the right of the zero and negative integers exist to the left of the zero.



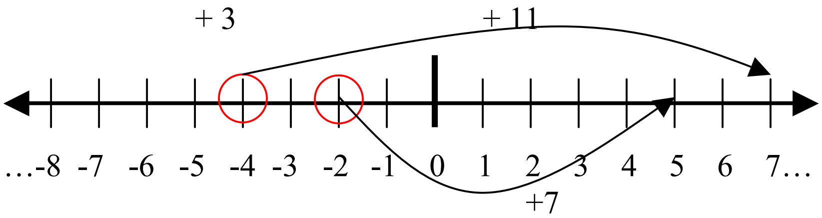
Adding Positive and Negative Integers

1) - 7 + 5 = -2
2) - 6 + 3 = -3
3) - 2 + 7 = 5
4) - 4 + 11 = 7

(1 & 2)



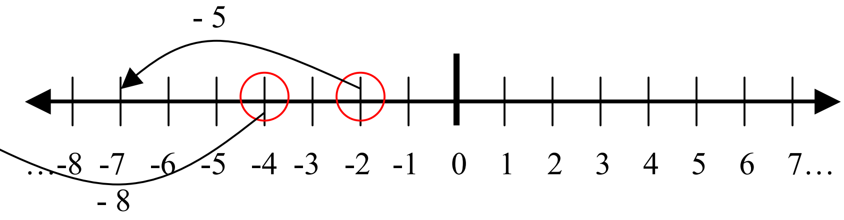
(3 & 4)



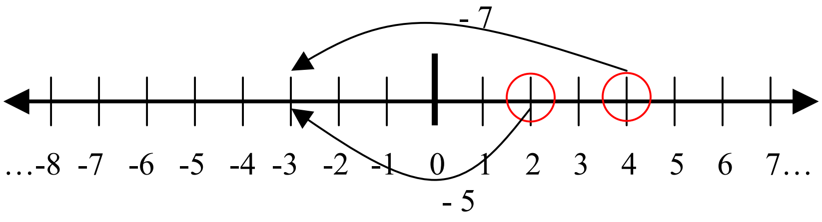
Subtracting Positive and Negative Integers

1) - 2 – 5 = -7
2) - 4 – 8 = -12
3) 4 – 7 = -3
4) 2 – 5 = -3

(1 & 2)



(3 & 4)



When adding and subtracting positive and negative integers you must know what to do when two signs are directly beside each other.

2 Positives	**2 Negatives**	**Opposite Signs**
+ + = +	- - = +	+ - = -

For instance:	6 + (+3)	6 + (-3)	6 - (-3)
	= 6 + 3	= 6 – 3	= 6 + 3
	= 9	= 3	= 9

Try these sample questions. The answers are below.

1) 5 – 9 =

2) – 4 + 6 =

3) – 5 – 2 =

4) 2 – 7 =

5) –2 + 5 =

6) 1 – 9 =

7) 4 – (+6) =

8) –2 –(-4) =

9) + 3 – (-6) =

10) 6 + (-4) =

11) 6 + (+2) =

12) -3 + (-2) =

Multiplying and Dividing Positive and Negative Integers

While multiplying and dividing positive and negative integers, remember the rules that apply to adding and subtracting integers with two signs directly beside each other.

2 Positives	**2 Negatives**	**Opposite Signs**
+ + = +	- - = +	+ - = -

You should break questions like this into two steps.

Step 1: Solve the equation ignoring the signs.

6 x (-3) = 18
5 x (-7) = 35
-12 ÷ (-4) = 3
36 ÷ (-9) = 4

- 5 x 4 = 20
- 3 x (-4) = 12
-21 ÷ 3 = 7
-64 ÷ (-8) = 8

If you ignored the + and – signs in front of the numbers you would end up with the answers above.

Step 2: Determine the + / - sign. The rules about + / - integers come into play. If there are two + signs, then the equation is positive. If there are two – signs, then the equation is also positive. If there is one + and one – sign, then the equation is negative.

6 x (-3) = **-18** (+ / -)   - 5 x 4 = **-20** (- / +)
5 x (-7) = - **35** (- / +)   - 3 x (-4) = **12** (- / -)
-12 ÷ (-4) = **3** (- / -)   -21 ÷ 3 = **- 7** (- / +)
36 ÷ (-9) = **- 4** (+ / -)   -64 ÷ (-8) = **8** (- / -)

The final answers are displayed in bold above.

Try these sample questions. The answers are posted below.

a) 3 x (– 6) =   b) – 2 x (-9) =   c) – 18 ÷ (–9) =

d) 7 x 7 =   e) –72 ÷ 8 =   f) –12 x (– 9) =

g) 7 x (-6) =   h) –28 ÷ (-4) =   i) 16 ÷ (-4) =

j) 3 x (-4) =   k) -45 ÷ (-15) =   l) -3 x (2) =

Answers to Sample Questions

1) 5 – 9 = **- 4**   2) – 4 + 6 = **2**   3) – 5 – 2 = **- 7**

4) 2 – 7 = **-5**   5) –2 + 5 = **3**   6) 1 – 9 = **-8**

7) 4 – (+6) = **-2**   8) –2 –(-4) = **2**   9) + 3 – (-6) = **9**

10) 6 + (-4) = **2**   11) 6 + (+2) = **8**   12) -3 + (-2) = **-5**

---

a) 3 x (– 6) = **-18**   b) – 2 x (-9) = **18**   c) – 18 ÷ (–9) = **2**

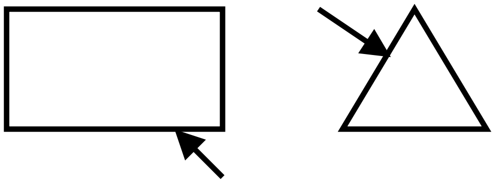
d) 7 x 7 = **49**   e) –72 ÷ 8 = **-9**   f) –12 x (– 9) = **108**

g) 7 x (-6) = **-42**   h) –28 ÷ (-4) = **7**   i) 16 ÷ (-4) = **-4**

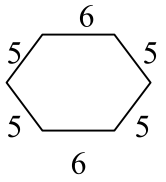
j) 3 x (-4) = **-12**   k) -45 ÷ (-15) = **3**   l) -3 x (2) = **-6**

Perimeters

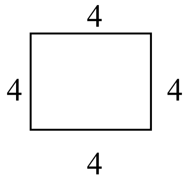
Perimeter is defined as the border around an object, or the outside edge of an object.



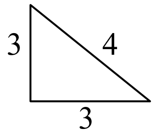
Perimeter is calculated by adding the sides of the object together.



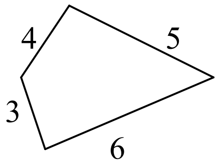
Perimeter = 6 + 5 + 5 + 5+ 5 + 6
= 32



Perimeter = 4 + 4 + 4 + 4
= 16



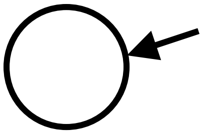
Perimeter = 3 + 3 + 4
= 10



Perimeter = 3 + 4 + 5 + 6
= 18

Circumferences

Circumference is also defined as the border around a shape, but is always associated with a circle.



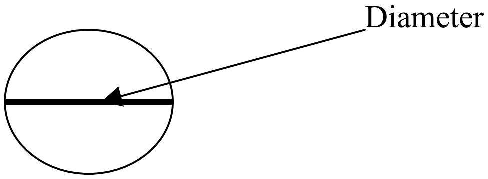
In order to determine the circumference of a circle, you must use a formula. You need to be familiar with some definitions.

π = 3.14 (pi)

You are going to have to remember that pi is equal to 3.14.

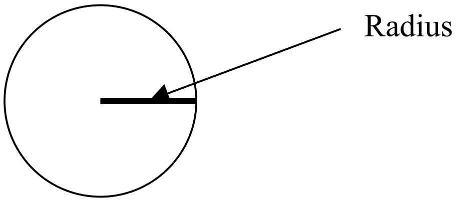
Diameter (d)

Diameter is the distance from one edge of the circle, through the middle, to the opposite side of the circle.



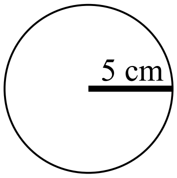
Radius (r)

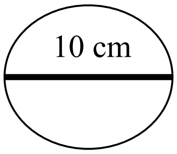
Radius is defined as ½ of the diameter, or the distance from the mid-point of a circle to its outer edge.



Formula for Calculating Circumference

$C = 2 \text{ x } (\pi) \text{ x } r$ or $C = d \text{ x } (\pi)$





$C = 2 \text{ x } (3.14) \text{ x } 5$

$= 31.4$ cm

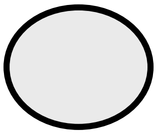
$C = 10 \text{ x } (3.14)$

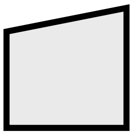
$= 31.4$ cm

The information you are given in a question will dictate the formula you should use to calculate the circumference. If you are given the radius, calculate the diameter by multiplying by two. Dividing the diameter by two will give you the radius.

Areas

Area is space that is occupied within the borders of a shape. It is measured in units squared and is represented by the area shaded in the shapes below.

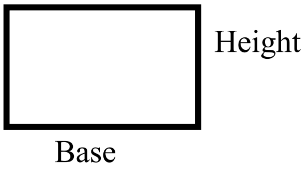


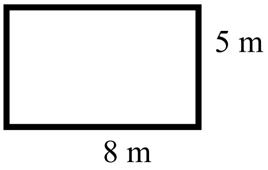


The three shapes you should know how to calculate area for are the triangle, rectangle and circle.

Area of a Rectangle or Square

To calculate the area of a square or rectangle, multiply the base of the object by its' height.

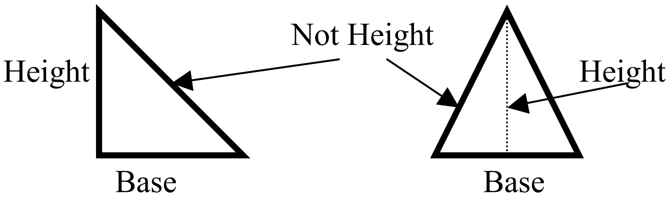


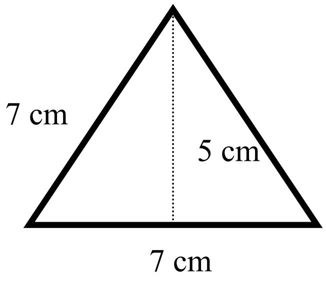


A = b x h This formula should be memorized
= 8 x 5
= 40 m^2

Area of a Triangle

To calculate the area of a triangle, follow the formula below.



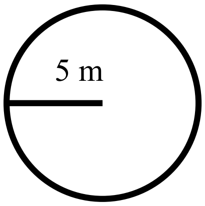


A = ½ x b x h **This formula should be memorized.**
= ½ x 7 x 5
= 17.5 cm ²

Remember that height is not necessarily an edge of the triangle, but the distance from the base to the top of the triangle.

Area of a Circle

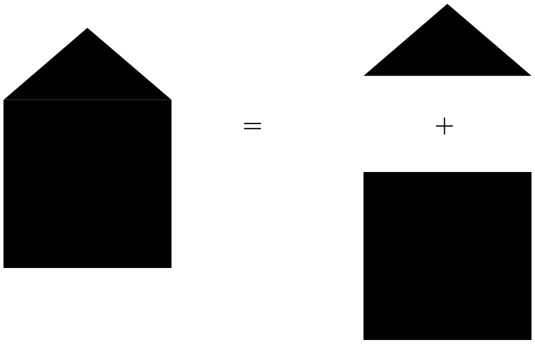
To calculate the area of a circle, follow the formula below.



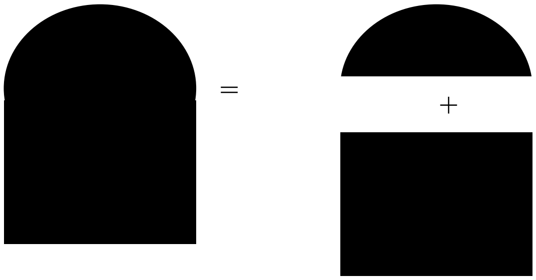
A = π (r) ² **This formula should be memorized.**
= (3.14) (5) ²
= (3.14) (25)
= 78.5 m ²

Other Shapes

You may have to calculate the area of shapes other than basic squares, triangles and circles. You can attempt to break shapes into smaller components and use the formulas above. For example:



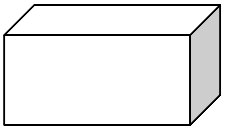
Calculate the area of the triangle and adding it to the area of the square results in the area of the whole shape.

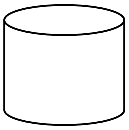


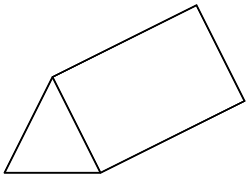
You can divide the shape on the left into a square and a half circle. Calculate the area of the square and the area of the circle. Divide the area of the circle in half and add the two together.

Volumes

Volume is defined as the area occupied by a three dimensional shape. If you pictured an empty cup, volume is the amount of liquid it contains. Calculating volume for different objects can be very difficult and involves complex formulas. We will discuss how to calculate the volume of three simple objects. Volume is always discussed in units cubed (example $3m^3$.)

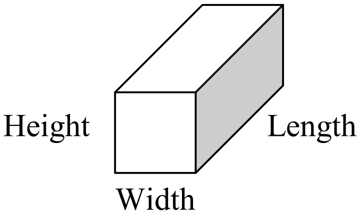


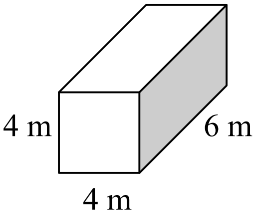




Volume of a Cube

You should memorize the formula for calculating the volume of a cube.





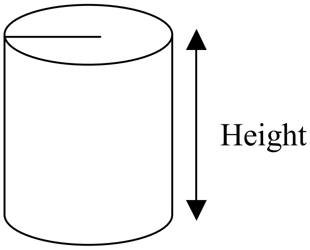
V = length x width x height
= 6 x 4 x 4
= 96 m^3

Volume of a Cylinder

To calculate the volume of a cylinder, determine the area of the circle and multiply it by the height of the cylinder.

Radius = 5 m

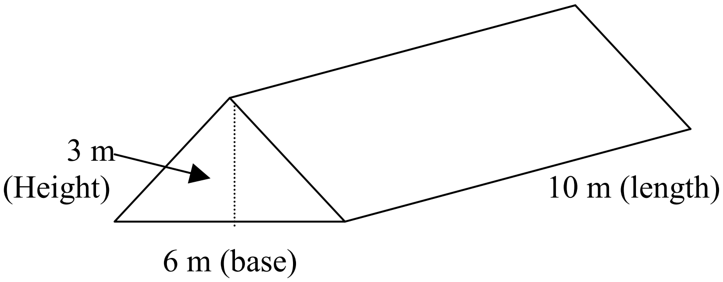
Height = 10 m



V = $\pi\ (r)^2$ x height
= $(3.14)\ (5)^2\ (10)$
= 785 m^3

Volume of a Triangular Shaped Object

To calculate the volume of an object like the one below, first calculate the area of the triangle and multiply it by the height of the object.



V = ½ (base) (height) (length)
= ½ (6) (3) (10)
= 90 m ^{3}

Metric Conversions

The key to understanding metric conversions is to memorize the prefixes and roots to each word. The root of each word indicates the basic measurement (litre, metre, gram), while the prefixes determine the relative size of the measurement (larger or smaller units – milli, centi, kilo, etc,).

Prefixes

All units in the metric system are easily converted because they are all based on units of 10. When converting between different measurements of the same base unit, it is as easy as shifting the decimal point.

For example:

432,000 millimetres
43,200 centimetres ALL EQUAL EACH OTHER
432 metres
0.432 kilometres

Length

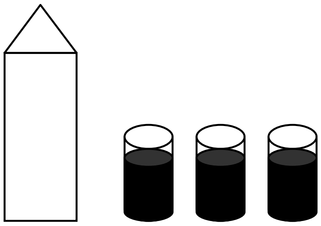
Length is used to measure the distance between points. The base unit for length is the metre. The most common units you'll encounter with length include:

Millimetres – small units (25 millimetres in 1 inch)
Centimetres – small units (2.5 centimetres in 1 inch)
Metres – larger units (1 metre = 3.2 feet or 1.1 yards)
Kilometres – large units (1.6 kilometres in 1 mile)

Prefix	**Example**	**Sign**	**Conversion**
Milli	Millimetres	mm	1 m = 1000 mm
Centi	Centimetre	cm	1 m = 100 cm
Deci	Decigram	dm	1 m = 10 dm
-	Metre	m	1 m = 1 m
Kilo	Kilometre	km	1 km = 1000 m

Volume

Volume is defined as the capacity of a given container. It usually measures the amount of liquid or gas that an object can hold. For example, the volume of a pop can is 355 millilitres, or the volume of a milk carton is 1 litre. The base unit for volume in the metric system is the litre. A litre is roughly the amount of milk that will fit into a milk carton or roughly three glasses of milk.



1 litre = 2.1 pints

1 gallon = 3.8 litres

The most common prefix used with volume is the millilitre (used to measure small amounts, such as tablespoons.) The majority of the time when measuring volume you will be using the litre measurement itself.

Prefix	**Example**	**Sign**	**Conversion**
Milli	Millilitres	mL	1 L = 1000 mL
Centi	Centilitres	cL	1 L = 100 cL
Deci	Decilitres	dL	1 L = 10 dL
-	Litres	L	1 L = 1 L
Kilo	Kilolitres	kL	1 kL = 1000 L

Mass or Weight

The base unit for weight in the metric system is the gram. The most common units you'll encounter with weight are:

Milligrams – very small (1000 milligrams in 1 gram)
Grams – small units (28.3 grams in 1 ounce)
Kilograms – large units (1 kilogram = 2.2 pounds)

Prefix	**Example**	**Sign**	**Conversion**
Milli	Milligrams	mg	1 g = 1000 mg
Centi	Centigram	cg	1 g = 100 cg
Deci	Decigram	dg	1 g = 10 dg
-	Gram	G	1 g = 1 g
Kilo	Kilogram	kg	1 kg = 1000 g

Algebraic Equations

Before beginning this section, make sure that you are comfortable with the rules of order of operation in mathematical equations. It is necessary to know in what order you add, subtract, divide and multiply in an equation.

Algebraic equations involve using letters and symbols to represent unknown numbers. In order to solve these equations you must isolate the unknown variable. We will begin with a couple of simple examples.

When solving algebraic equations, it is important to know the opposite mathematical operations. For example, subtraction is the opposite of addition and division is the opposite of multiplication. Square roots are the opposite of squaring. We will not cover square roots in this section.

$6 + y = 12$

$6 + y \mathbf{- 6} = 12 \mathbf{- 6}$

$y = 6$

In order to isolate the “y”, eliminate a + 6 on the left hand side of the equation. In algebraic equations, whatever you do to one side of the equation you must also do to the other side. Subtract 6 from both sides.

$y - 3 = 15$

$y - 3 \mathbf{+ 3} = 15 \mathbf{+ 3}$

$y = 18$

In order to isolate the “y”, eliminate a - 3 on the left hand side of the equation. Add 3 to both sides.

$7\ y = 42$

$7\ y / \mathbf{7} = 42 / \mathbf{7}$

$y = 6$

In this case, “y” is multiplied by 7. To eliminate a number that is being multiplied, divide by the same number. Divide both sides by 7.

$y / 12 = 5$

$y / 12\ \mathbf{x\ 12} = 5\ \mathbf{x\ 12}$

$y = 60$

In this case, “y” is divided by 12. To eliminate a number that is being divided, multiply by the same number. Multiply both sides by 12.

Practice solving some of these simple equations:

1) $y / 11 = 23$
2) $15 + y = 63$
3) $-5 + y = 10$
4) $13 (y) = 130$
5) $5 y = 15$
6) $6 + 3 + y = 56$
7) $2(y) = 56$
8) $y / 8 = 4$
9) $y (24) = 72$

Answers are below.

More Advanced Algebraic Equations

When solving equations, follow the order of operations which dictate that you perform equations within brackets, followed by exponents, then division and multiplication, and finally addition and subtraction. When isolating unknown variables, use the opposite order. We will not cover solving equations with exponents at this level.

$6 y + 12 = 84$

$6 y + 12 \mathbf{- 12} = 84 \mathbf{- 12}$

$6 y = 72$

$6 y \mathbf{/ 6} = 72 \mathbf{/ 6}$

$y = 12$

In order to isolate the "y", first eliminate a + 12 on the left hand side of the equation. Subtract 12 from both sides. You are left with 6y = 12.
To isolate "y", now simply divide both sides of the equation by 6.

$y / 3 + 12 - 2 = \mathbf{15 \times 3} + 4$

$y / 3 + 12 - 2 = 45 \mathbf{+ 4}$

$y / 3 + 12 - 2 = 49$

$y / 3 + 12 - 2 \mathbf{+ 2} = 49 \mathbf{+ 2}$

$y / 3 + 12 = 51$

$y / 3 + 12 \mathbf{- 12} = 51 \mathbf{- 12}$

$y / 3 = 39$

$y / 3 \mathbf{\times 3} = 39 \mathbf{\times 3}$

$y = 117$

You may encounter equations where one side has operations without an unknown variable. In cases like this, solve the side without an unknown variable FOLLOWING THE STANDARD ORDER OF OPERATION RULES.

After you have accomplished this, solve the equation in the standard manner. People more advanced in math will be able to consolidate portions of the left side as well, but unless you are comfortable you should proceed the way outlined to the left.

(6 – y) x 3 = 24

(6 – y) x 3 / **3** = 24 / **3**

(6 – y) = 8

6 – y **– 6** = 8 **- 6**

- y = 2

- y **x (-1)** = 2 **x (-1)**

y = - 2

Perform this equation following the standard rules. Leave the brackets until the end. When only the brackets remain, you can get rid of them as they no longer serve a purpose.

When you are left with an equation where the unknown is isolated, but negative, simply multiply both sides of the equation by – 1 to inverse the signs.

The end result is that y = - 2.

18 / y = 2

18 / y **x (y)** = 2 **x (y)**

18 = 2 y

18 **/ 2** = 2 y / **2**

9 = y

One other tricky situation you may encounter is when “y” appears on the bottom of a division equation. In order to solve for “y”, move it from the bottom of the division sign by multiplying both sides of the equation by “y”. The result is 18 = 2 y. Now solve the rest of the equation.

WHATEVER YOU DO TO ONE SIDE OF AN EQUATION YOU MUST ALSO DO TO THE OTHER SIDE.

More Practice Questions

a) 3 (y) + 6 – 10 = 89

b) (y) / 6 + 24 – 2 = 14

c) - y (3) +55 = 105

d) 5 y – 32 = 24 (3)

e) –32 + 6y/2 = 64

f) 22 y + 16 (8) = 6 y

Answers:

1) 253 2) 48 3) 15
4) 10 5) 3 6) 47
7) 28 8) 32 9) 3

a) 31 b) - 48 c) –16.7
d) 20.8 e) 32 f) – 8